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LONGITUDINAL IMPACT OF TWO MUTUALLY PLASTICALLY-DEFORMABLE MISSILES

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Abstract—This paper develops an improved theory of large plastic deformation of metallic cylindrical flat ended missiles striking each other with prescribed launch velocities and compares results with experimental data.

The groups of comparison studies which have been used for comparison with this analysis are:

(a) G. I. Taylor and A. C. Whiffin
(b) E. H. Lee, S. T. Tupper, A. C. Whiffin and H. L. D. Pugh
(c) N. Davids, C. Graberek, D. Raftopoulos and A. Ricchiazzi

It has been found from the above comparisons that the results given in this paper show that the theory predicts large deformation of the aforesaid missiles satisfactorily for low and intermediate velocities.

NOTATION*

A_0	initial, undeformed cross-sectional area
A_j	variable cross-sectional area
c_e	elastic wave velocity
c_j	relative wave front velocity
c_w	absolute velocity of plastic boundary
d	diameter
$\mathrm{d}m$	mass element undergoing plastic deformation during $\mathrm{d}t$
$\mathrm{d}M$	mass of the undisturbed part
$\mathrm{d}t$	time interval
$\mathrm{d}v$	arbitrary velocity increment
$\mathrm{d}v_1$	change of velocity in rear part
$\mathrm{d}X$	undisturbed length
F	reaction force
f	net force on element
j	index, successive states
l	length of missile
t	time
v	particle velocity
v_1	impact velocity
v^*	common velocity of the plastically-deformed part
x^*	longitudinal coordinate of plane dividing the missiles
x_j	longitudinal coordinate of rigid–plastic boundary
x_1	coordinate of left end of missile
ε	axial strain
ρ	mass density of missile material
σ	stress
σ_y	yield-stress magnitude

1. INTRODUCTION

THE purpose of this investigation is to study the deformation, motion and material behavior of metallic missiles when they strike each other with known launch velocities.

* Superscripts are used to identify symbols; l denotes quantities of left missile, while r denotes quantities of right missile.

Specifically, two main problems were investigated. First, cylindrical flat-ended missiles, of equal cross section and equal mass, strike each other with prescribed launch velocities, undergoing large plastic deformation according to some assumed material constitutive behavior. The analysis of this investigation is based on assumptions that Sir Geoffrey I. Taylor [1] introduced for flat-ended projectiles striking rigid targets. Then the developed theory is extended to be applied to missiles having unequal masses.

The method of approach which has been used for this problem is the one which is known as "Direct Analysis." It is the method in which the analysts do not derive the governing differential equations, but use the physical laws, the assumed material behavior, the kinematical relations, etc., directly in a numerical computer approach, from which results are acquired.

The results calculated in this work compare favourably with experimental data of Refs. [2–5].

2. DESCRIPTION OF THE PROBLEM AND ASSUMPTIONS

In the analysis of this problem the following theoretical model is assumed.

When two cylindrical missiles of equal cross section strike each other at normal obliquity, it is assumed that the parts of the missiles which are in contact or are close to the contact surface shall be permanently plastically deformed, while the rear parts of the missiles shall remain rigid. This model is pictured in Fig. 1(A) and 1(B). Figure 1(A) shows

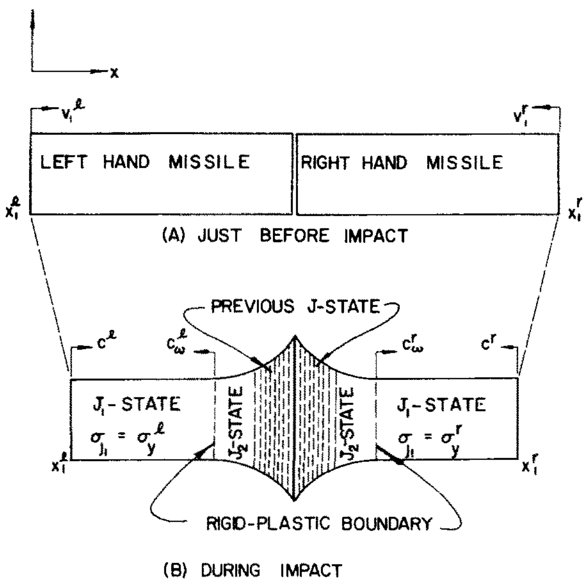

FIG. 1. Cylindrical missiles striking each other.

two cylindrical missiles traveling with impact velocities v_1^l and v_1^r just before they strike each other. Figure 1(B) depicts the missiles during their collision. It can be seen in this figure that each missile is divided into two main parts:

(a) rigid part

(b) permanently plastically-deformed part.

The two parts are separated by the plastic wave front (rigid–plastic boundary) which travels during the process of collision toward the rigid parts of the missiles. To develop simple theory which will deal with deformation of our model, the following assumptions were found necessary. The missiles are assumed to be of like material which exhibits a rigid, perfectly–plastic behavior. This assumption allows the elastic strains, small in comparison with the plastic ones, to be neglected, and also allows the rear part of the missiles to be treated as undeformed during the process of impact. It is also assumed that density remains constant in the plastic region, and, in order to deal with one-dimensional theory, radial inertia is neglected.

3. PHYSICAL LAWS

A. *Mass balance across plastic front*

By denoting $c^l_{j_1}$ and $c^r_{j_1}$ as the velocities of the plastic fronts relative to observers moving with the particles in the j_1 states of the left and right missile respectively, and v^* as the common velocity of the plastically-deformed part (see Figs. 2 and 3(A)), relations are arrived at which introduce transformation and refer to velocities relative to the motions of the j_1 states of the missiles.

$$c^l_{j_1} = c^l_w - v^l_{j_1} + v^* \qquad c^r_{j_1} = c^r_w - v^r_{j_1} + v^*. \tag{3.1}$$

The plastic wave fronts moving from X to X′ and Y to Y′, will advance in the media j_1 and j_2 the distances respectively (see Fig. 2)

$$\mathrm{d}x^l_1 = c^l_{j_1}\,\mathrm{d}t, \quad \mathrm{d}x^l_2 = c^l_{j_2}\,\mathrm{d}t \qquad \mathrm{d}x^r_1 = c^r_{j_1}\,\mathrm{d}t, \quad \mathrm{d}x^r_2 = c^r_{j_2}\,\mathrm{d}t. \tag{3.2}$$

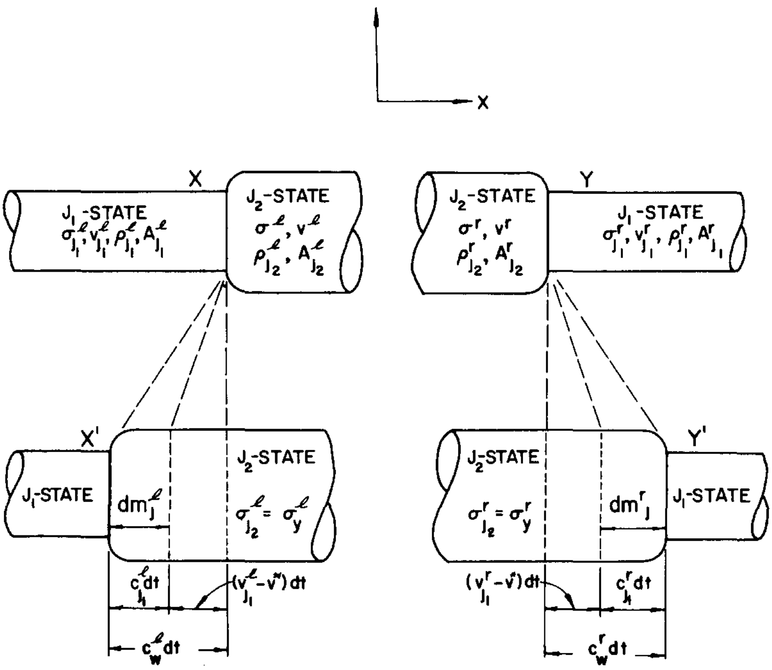

FIG. 2. Moving plastic fronts between states.

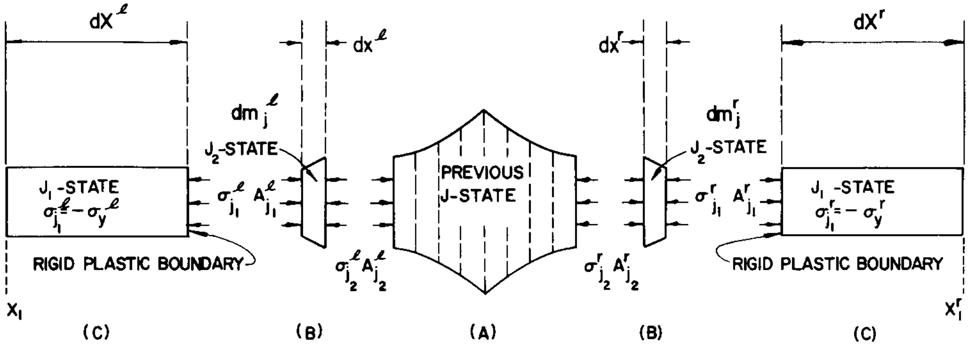

FIG. 3. (A) Deformed material (previous j-state); (B) Material elements being deformed during $\mathrm{d}t$; (C) Free body diagrams of rigid parts.

Therefore, the mass changes in the j_1 and j_2 states can now be expressed as

$$\begin{aligned} \mathrm{d}m_{j_1}^l &= -\rho_{j_1}^l A_{j_1}^l \,\mathrm{d}x_1^l \qquad & \mathrm{d}m_{j_1}^r &= -\rho_{j_1}^r A_{j_1}^r \,\mathrm{d}x_1^r \\ \mathrm{d}m_{j_2}^l &= \rho_{j_2}^l A_{j_2}^l \,\mathrm{d}x_2^l \qquad & \mathrm{d}m_{j_2}^r &= \rho_{j_2}^r A_{j_2}^r \,\mathrm{d}x_2^r . \end{aligned} \tag{3.3}$$

By applying the laws of constant density and conservation of mass, $\mathrm{d}m_{j_1} + \mathrm{d}m_{j_2} = 0$, and substituting (3.1) and (3.2) into (3.3), we derive expressions for the absolute speeds of the plastic wave fronts. Thus

$$\begin{aligned} c_w^l &= (v_{j_1}^l - v^*) + A_{j_2}^l (v_{j_1}^l - v_{j_2}^l)/(A_{j_1}^l - A_{j_2}^l) \\ c_w^r &= (v_{j_1}^r - v^*) + A_{j_2}^r (v_{j_1}^r - v_{j_2}^r)/(A_{j_1}^r - A_{j_2}^r). \end{aligned} \tag{3.4}$$

B. *Momentum balance*

Due to the motion of the plastic fronts, material is traversed during an incremental time $\mathrm{d}t$ and defines elements as shown in Fig. 3(B). The net forces are then calculated as in Ref. [2].

$$f^l = \mathrm{d}m_{j_1}^l (v_{j_2}^l - v_{j_1}^l)/\mathrm{d}t \qquad f^r = \mathrm{d}m_{j_1}^r (v_{j_2}^r - v_{j_1}^r)/\mathrm{d}t. \tag{3.5}$$

However, the net forces and the areas are related to the stresses as follows

$$f^l = -\sigma_{j_1}^l A_{j_1}^l + \sigma_{j_2}^l A_{j_2}^l \qquad f^r = \sigma_{j_1}^r A_{j_1}^r + \sigma_{j_2}^r A_{j_2}^r . \tag{3.6}$$

Solving these equations with respect to the unknown areas yields

$$A_{j_2}^l = (-f^l + \sigma_{j_1}^l A_{j_1}^l)/\sigma_{j_2}^l \qquad A_{j_2}^r = (f^r + \sigma_{j_1}^r A_{j_1}^r)/\sigma_{j_2}^r . \tag{3.7}$$

Since

$$\sigma_{j_1}^l = \sigma_{j_2}^l = \sigma_y \qquad \sigma_{j_1}^r = \sigma_{j_2}^r = \sigma_y^r$$

therefore

$$A_{j_2}^l = -f^l/\sigma_y^l + A_{j_1}^l \qquad A_{j_2}^r = f_{j_2}^r/\sigma_y^r + A_{j_1}^r . \tag{3.8}$$

The reaction force at the boundary is given by

$$F^l = -f^l + \sigma_y^l A_{j_1}^l \qquad F^r = f^r + \sigma_y^r A_{j_1}^r . \tag{3.9}$$

C. *Strain–area relation*

The strains in the deformed material for an incompressible material are

$$\varepsilon^l_{j_2} = A^l_0/A^l_{j_2} - 1 \qquad \varepsilon^r_{j_2} = A^r_0/A^r_{j_2} - 1. \tag{3.10}$$

D. *Rear part of missiles, impulse momentum law*

As in Ref. [2] the rigid parts of missiles (see also Fig. 3(C)) are in state j_1 and are subjected to forces

$$f^{*l} = \sigma^l_{j_1} A^l_{j_1} \qquad f^{*r} = \sigma^r_{j_1} A^r_{j_1}. \tag{3.11}$$

During an incremental time dt the missile's ends x^l_1 and x^r_1 shall move the distances $v^l_1\, dt$ and $v^r_1\, dt$. Therefore, the new positions $x^{l'}_1$ and $x^{r'}_1$ are located

$$x^{l'}_1 = x^l_1 + v^l_1\, \mathrm{d}t \qquad x^{r'}_1 = x^r_1 + v^r_1\, \mathrm{d}t. \tag{3.12}$$

The length of the rigid parts are

$$\mathrm{d}X^l = x^l_j - x^{l'}_1 \qquad \mathrm{d}X^r = x^r_j - x^{r'}_1 \tag{3.13}$$

and their masses are then calculated

$$\mathrm{d}M^l = \rho^l_{j_1} A^l_{j_1}\, \mathrm{d}X^l \qquad \mathrm{d}M^r = \rho^r_{j_1} A^r_{j_1}\, \mathrm{d}X^r. \tag{3.14}$$

By applying the impulse–momentum law

$$\mathrm{d}v^l_1 = \sigma^l_{j_1} A^l_{j_1}/\mathrm{d}M^l \qquad \mathrm{d}v^r_1 = \sigma^r_{j_1} A^r_{j_1}/\mathrm{d}M^r \tag{3.15}$$

and the impact velocities are then reduced to

$$v^{l'}_1 = v^l_1 + \mathrm{d}v^l_1\, \mathrm{d}t \qquad v^{r'}_1 = v^r_1 + \mathrm{d}v^r_1\, \mathrm{d}t. \tag{3.16}$$

The new coordinates of the plastic boundaries are calculated by using the relations

$$x^{l'}_j = x^l_j + (c^l_w + v^*)\, \mathrm{d}t \qquad x^{r'}_j = x^r_j + (c^r_w + v^*)\, \mathrm{d}t \quad \text{for} \quad j = 2, 3, \ldots \tag{3.17}$$

and the coordinates of the plane dividing the two missiles are computed by

$$x^{*'} = x^* + v^*\, \mathrm{d}t. \tag{3.18}$$

4. METHOD OF ANALYSIS

The Direct Analysis method has been used to solve this problem. A discussion of this method has been given in Refs. [6] and [7] and therefore will not be repeated. The process begins by specifying the constants, the initial conditions, and marking out the passage of time discretely. Then, the ground state of the missiles is defined, viz.

$$\begin{aligned} \sigma^r_j &= \sigma^l_j = \sigma_y \qquad & A^r_j &= A^l_j = A_0 \\ v^r_j &= v^r_1 & v^l_j &= v^l_1 \qquad j = 1 \end{aligned}$$

where

$$v^r_1 > 0 \qquad v^l_1 < 0.$$

During the first incremental time dt the plastic wave fronts define two elements as shown in Fig. 3(B). For these elements, one of the dynamical variables is decreased and the levels or contours are moved through the elements as a function of time. The variable used for this purpose may be arbitrarily chosen, e.g. strain, stress or velocity. In this paper a sequence of decreasing velocities was introduced as independent variables.

If v_j^l and v_j^r are current velocities of missiles, then

$$v_{j+1}^l = v_j^l + dv^l \qquad v_{j+1}^r = v_j^r + dv^r$$

where $dv^l < 0$ and $dv^r > 0$ ($|dv^r| = |dv^l|$) are arbitrarily specified negative and positive increments respectively, which will reduce the magnitude of the velocities of these elements. Ultimately as the index j increases, the velocities will be approximately the same, depending on the coarse increment dv^r and dv^l ($v_j^l = v_j^r = v^*$ boundary condition), for both elements. When this happens the loading is terminated for this incremental time.

Actually two boundary conditions are to be satisfied at the interface, continuity of velocity and continuity of reaction forces. The first condition, continuity of velocity, has been discussed. The second condition, continuity of reaction forces, needs no special arrangement in order to be satisfied for the case of missiles of equal cross sections and like materials which are concerned in this paper. By investigating equations (3.6)–(3.9) one can see that the magnitude of F^l is equal to the magnitude of F^r. During this process the other dynamical variables (being related directly to the current velocities) are also calculated by using the relations (3.1) to (3.18). This completes all the calculations at time t. The next step is to increase the time by dt and repeat the calculations cyclically. The motion and the process will be terminated when the velocities of the rigid parts of the missiles (which are computed for every incremental time by using equation (3.16)) are equal, i.e. $v_1^{l'} = v_1^{r'}$.

In the paragraph below the basic scheme of the method is listed. State j_1 is the ground state ($j_1 = j = 1$), and state j_2 is the state of the portion of plastically-deformed material which is adjacent to the plastic boundary; $j_2 = j+1$ where j is the current value of the index.

Basic scheme of the method	Remarks
1. $l^l, l^r, d^l, d^r, \sigma_y^l, \sigma_y^r, v_1^l, v_1^r, dv^l, dv^r$ $k_m, dt, \rho_0^l, \rho_0^r$	Given data
2. $m^l, m^r, A_0^l, A_0^r, x_1^l, x_1^r$	From given data
3. $\varepsilon_j^l = \varepsilon_j^r = v_j^l = v_j^r = x_j^l = x_j^r = 0$ $c_j^l = c_j^r = c_{w_j}^l = c_{w_j}^r = 0$	Initialization of all quantities
4. $\sigma_j^l = \sigma_y^l, \sigma_j^r = \sigma_y^r, A_j^l = A_0^l, A_j^r = A_0^r$ $\rho_j^l = \rho_0^l, \rho_j^r = \rho_0^r$	Specification of ground state
5. $v_j^l = v_1^l, v_j^r = v_1^r$ $v_{j+1}^l = v_j^l + dv^l$; $v_{j+1}^r = v_j^r + dv^r$	Specification of velocities of missiles velocities, levels
6. $c_j^l = c_w^l - v_1^i + v^*$; $c_j^r = c_w^r - v_1^r + v^*$	Law for velocity of plastic fronts

Basic scheme of the method	Remarks
7. $dx^l = c_j^l\, dt$; $dx^r = c_j^r\, dt$	Calculation of incremental displacement of plastic front
8. $dm_j^l = \rho_j^l A_j^l\, dx_1^l$; $dm_j^r = \rho_j^r A_j^r\, dx_1^r$	Mass change of j-state
9. $c_{w_j}^l = [A_{j+1}^l(v_{j+1}^l - v_j^l)/(A_{j+1}^l - A_j^l)] + v_1^l - v^*$ $c_{w_j}^r = [A_{j+1}^r(v_{j+1}^r - v_j^r)/(A_{j+1}^r - A_j^r)] + v_1^r - v^*$	Absolute wave speed
10. $\varepsilon_{j+1}^l = A_0^l/A_{j+1}^l - 1$; $\varepsilon_{j+1}^r = A_0^r/A_{j+1}^r - 1$	Strain–area law
11. $f^l = (v_{j+1}^l - v_j^l)\, dm_j^l/dt$; $f^r = (v_{j+1}^r - v_j^r)\, dm_j^r/dt$	Net forces on elements
12. $A_{j+1}^l = (-f^l + \sigma_j^l A_j^l)/\sigma_{j+1}^l$; $A_{j+1}^r = (f^r + \sigma_j^r A_j^r)/\sigma_{j+1}^r$	New areas (plastic state)
13. $F^l = \sigma_{j+1}^l A_{j+1}^l$; $F^r = -\sigma_{j+1}^r A_{j+1}^r$	Reaction forces of missiles
14. if $v_j^l - v_j^r \begin{bmatrix} \leq 0 \text{ end loading} \\ > 0 \text{ continue loading} \end{bmatrix}$	Test for termination of loading
15. $x_1^l = x_1^l + v_1^l\, dt$; $x_1^r = x_1^r + v_1^r\, dt$	Displacements of free ends
16. $dX^l = x_j^l - x_1^l$; $dX^r = x_j^r - x_1^r$	Lengths of undisturbed parts
17. $dM^l = \rho_j^l A_j^l\, dX^l$; $dM^r = \rho_j^r A_j^r\, dX^r$	Masses of undisturbed parts
18. $dv_1^l = \sigma_j^l A_j^l\, dt/dM^l$; $dv_1^r = \sigma_j^r A_j^r\, dt/dM^r$	Impulse momentum, rear parts
19. $v_1^l = v_1^l + dv_1^l$; $v_1^r = v_1^r + dv_1^r$	New velocities of rear parts
20. $x_j^l = x_j^l + (c_{w_j}^l + v^*)\, dt$; $x_j^r = x_j^r + (c_{w_j}^r + v^*)\, dt$ for $j = 2 \ldots j_m$	Displacement of plastic boundary
21. $x^* = x^* + v^*\, dt$	Displacement of plane dividing two missiles
22. if $v_1^l - v_1^r \begin{bmatrix} \leq 0 \text{ end impact} \\ > 0 \text{ continue impact} \end{bmatrix}$	Test for end of impact

The time t is then increased by dt and we repeat the calculations cyclically.

5. RESULTS

Some thought has been given by experimenters to the development of a mutual missile collision experimental arrangement. The practical difficulties are severe, because of problems of timing and alignment. The benefits, however, are attractive because relative impact velocities are greater by such an arrangement.

No direct experimental data are at present available for comparison here. However, when the velocities and the masses of the missiles are equal to each other, the deformation of each missile should be exactly the same as if it alone had impacted a rigid target.

Therefore, it was decided to compare the results of this analysis with the available experimental data of projectiles striking rigid targets. Figures 4 and 5 show Taylor–Whiffin and Lee–Tupper projectiles, respectively, with different striking velocities as they would appear in the case of a mutual impact. The dynamic yield stress for the Taylor and Whiffin projectile was found to vary from 140,000 psi for impact velocity 1120 ft/sec to 180,000 psi for impact velocity 2120 ft/sec. In the above comparisons the deformation of projectiles due to the target deformation was eliminated. For the Lee and Tupper projectile, the dynamic yield stress was found to be the same (282,000 psi) for all different impact velocities (see Ref. [4]). The validity of the assumption that radial inertia is negligible has been verified by the following comparison with the experimental data of Ballistic Research Laboratories.

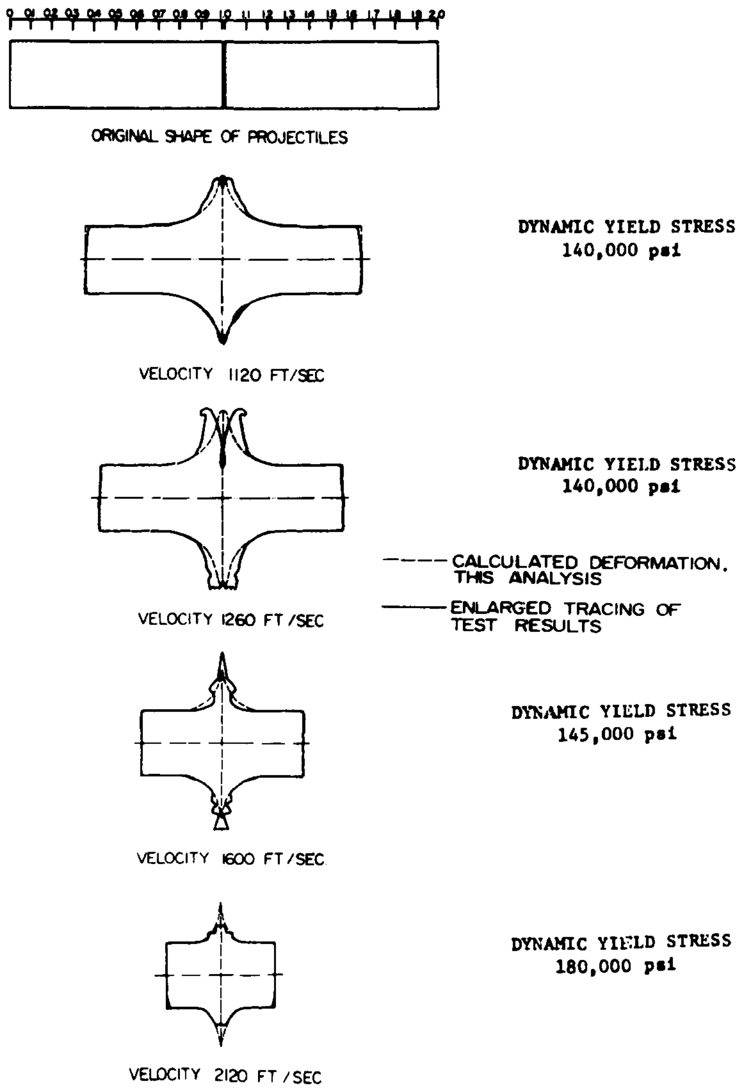

FIG. 4. Collision of two deformable cylinders of equal mass and equal impact velocity.

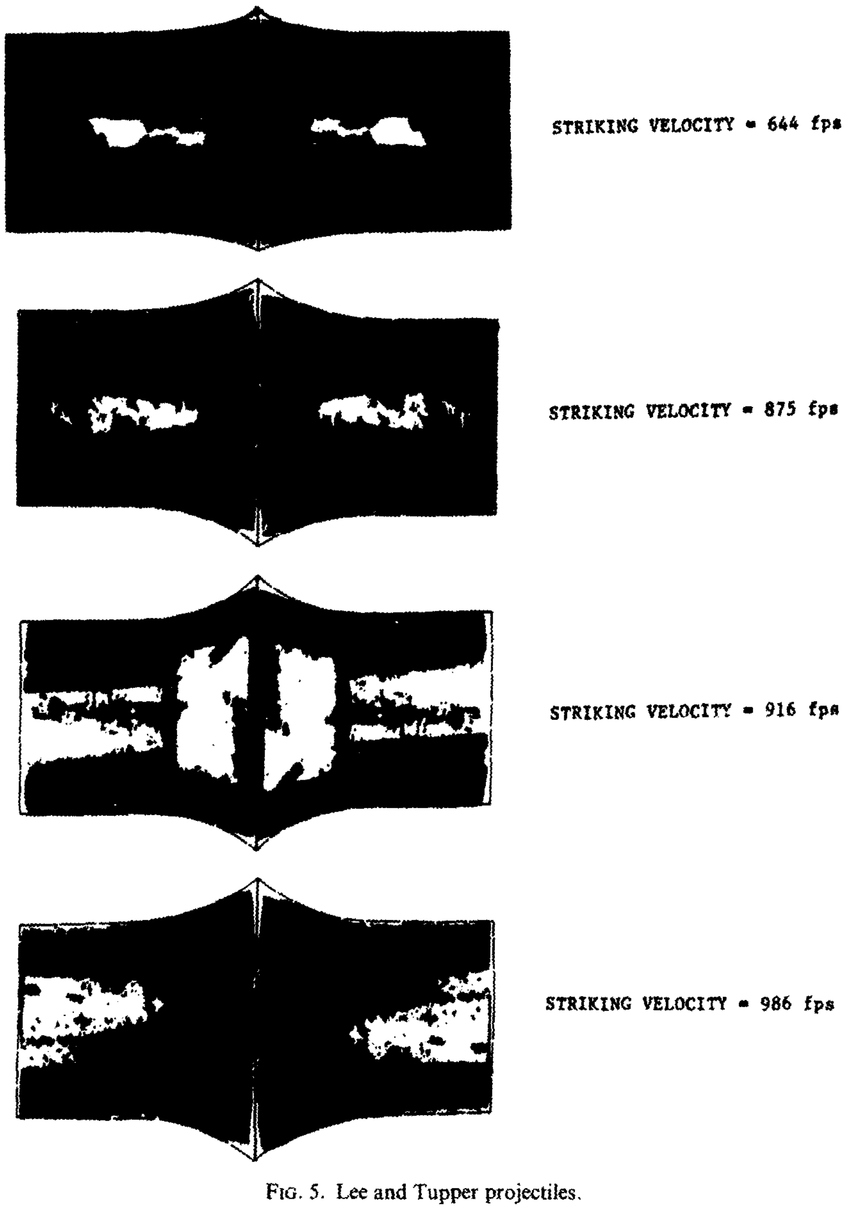

FIG. 5. Lee and Tupper projectiles.

In this experiment, Graberek and Ricchiazzi [5] fired cylindrical slugs, 1·613 in. dia. and 6 in. long, at normal obliquity against an armor target. Figure 6 shows these slugs compared with theoretical data of this analysis as they will appear in mutual impact. The dynamic yield stress was found to be 120,000 psi for these cylindrical slugs.

Figure 7 shows the calculated results of the analysis for projectiles which have equal velocities, and masses in the ratio ranging from 1 : 1 to 1 : 7. In the absence of experimental data, a dynamic yield stress of 140,000 psi was assumed to be valid for all these cases. "Residual velocity" is the velocity of both missiles at the termination of impact.

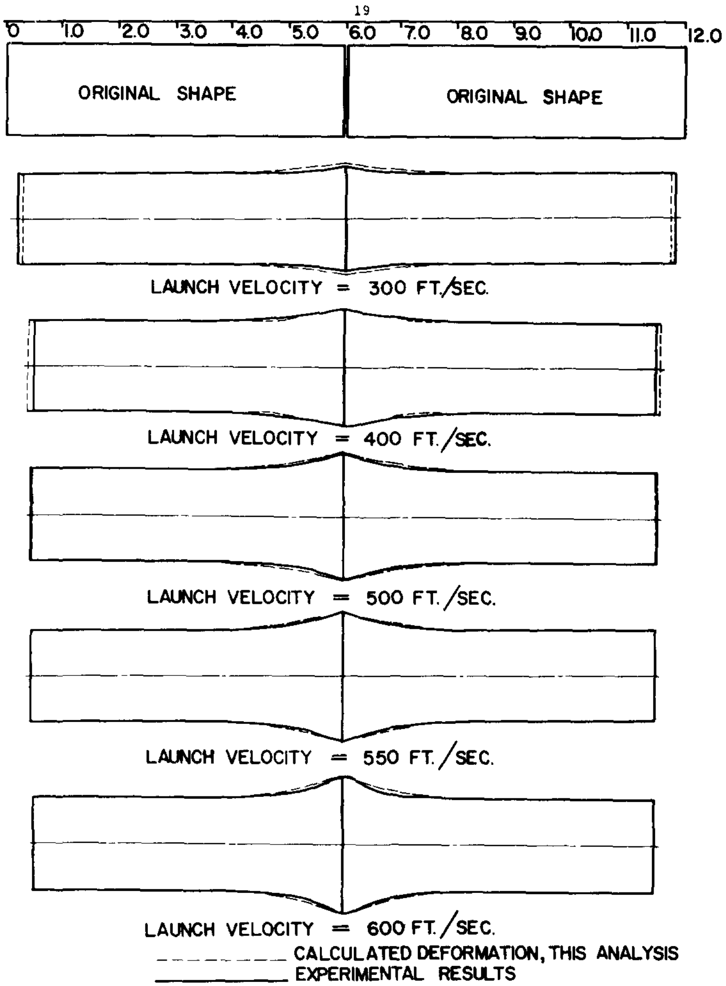

FIG. 6. BRL cylindrical slugs after test.

Figures 8 and 9 are X–T diagrams showing the motion of (a) free ends of missiles, (b) rigid plastic boundaries, and (c) plastically-deformed part of the missiles for the cases of mass ratio 1 : 2 and 1 : 7 respectively.

6. CONCLUSIONS

In this investigation a theoretical analysis was developed to determine the dynamic yield stress, the deformation, and the kinematics of missiles in mutual impact. The results of this analysis have been compared with experimental data for projectiles striking rigid targets. This comparison was made by noting that when two missiles of the same mass and the same velocity impact each other, the deformation of the missiles during and after impact, must be the same as in the case when each of the missiles strikes a rigid target. Three different experimental data have been used for comparison with the results of this analysis.

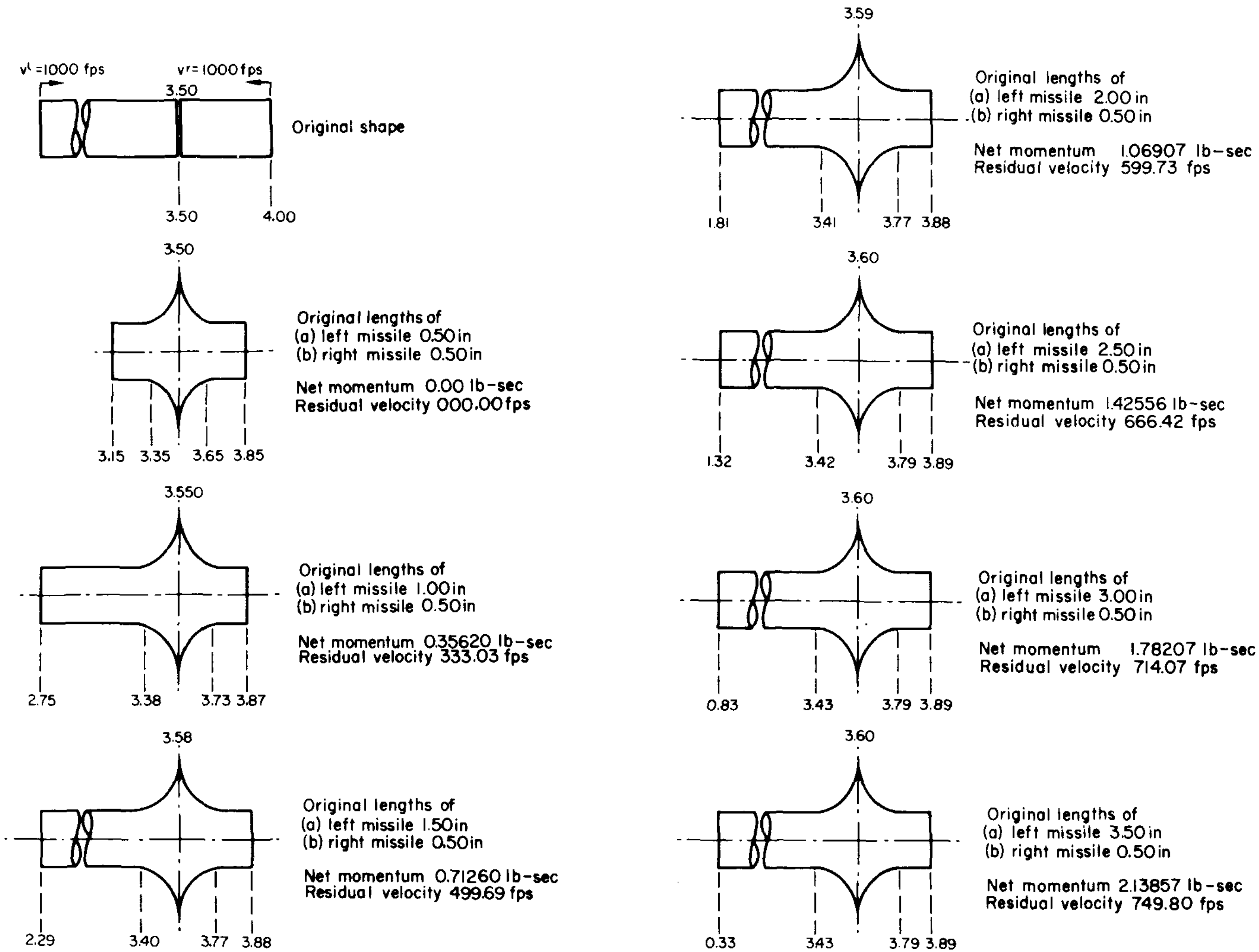

FIG. 7. Theoretical results, this analysis, for missiles of unequal masses.

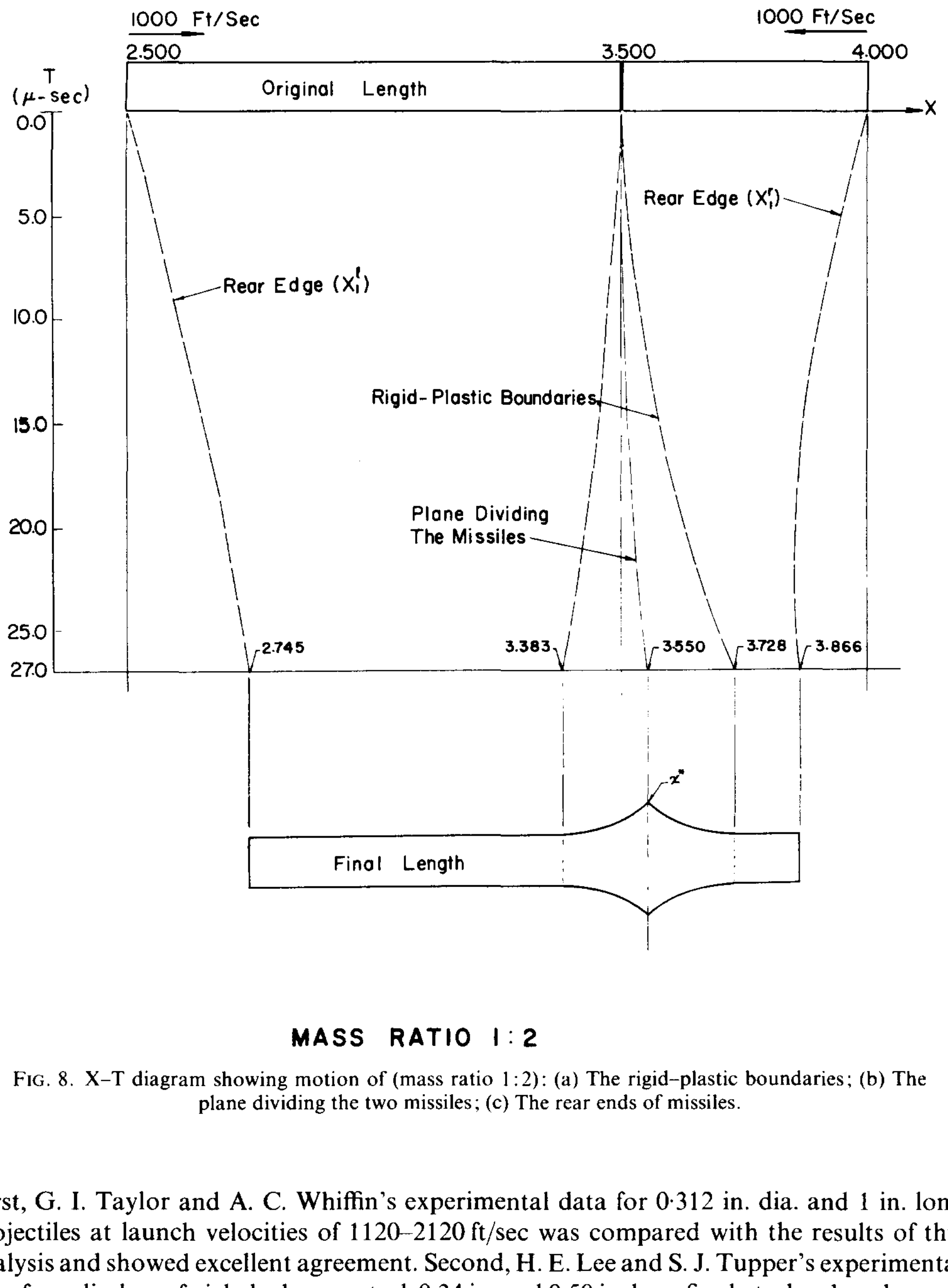

FIG. 8. X–T diagram showing motion of (mass ratio 1:2): (a) The rigid–plastic boundaries; (b) The plane dividing the two missiles; (c) The rear ends of missiles.

First, G. I. Taylor and A. C. Whiffin's experimental data for 0·312 in. dia. and 1 in. long projectiles at launch velocities of 1120–2120 ft/sec was compared with the results of this analysis and showed excellent agreement. Second, H. E. Lee and S. J. Tupper's experimental data for cylinders of nickel–chrome steel, 0·34 in. and 0·50 in. long fired at a hardened armor plate was compared with results of this analysis, and it was found that the dynamic influence on this material was very small. Third, some recent experimental data of BRL for large diameter cylinders, 1·613 in. dia. and 6 in. long, was compared with this analysis and has verified that the assumption of one-dimensional motion in plastic impact in this analysis gave remarkable agreement even in the case of large-diameter slugs.

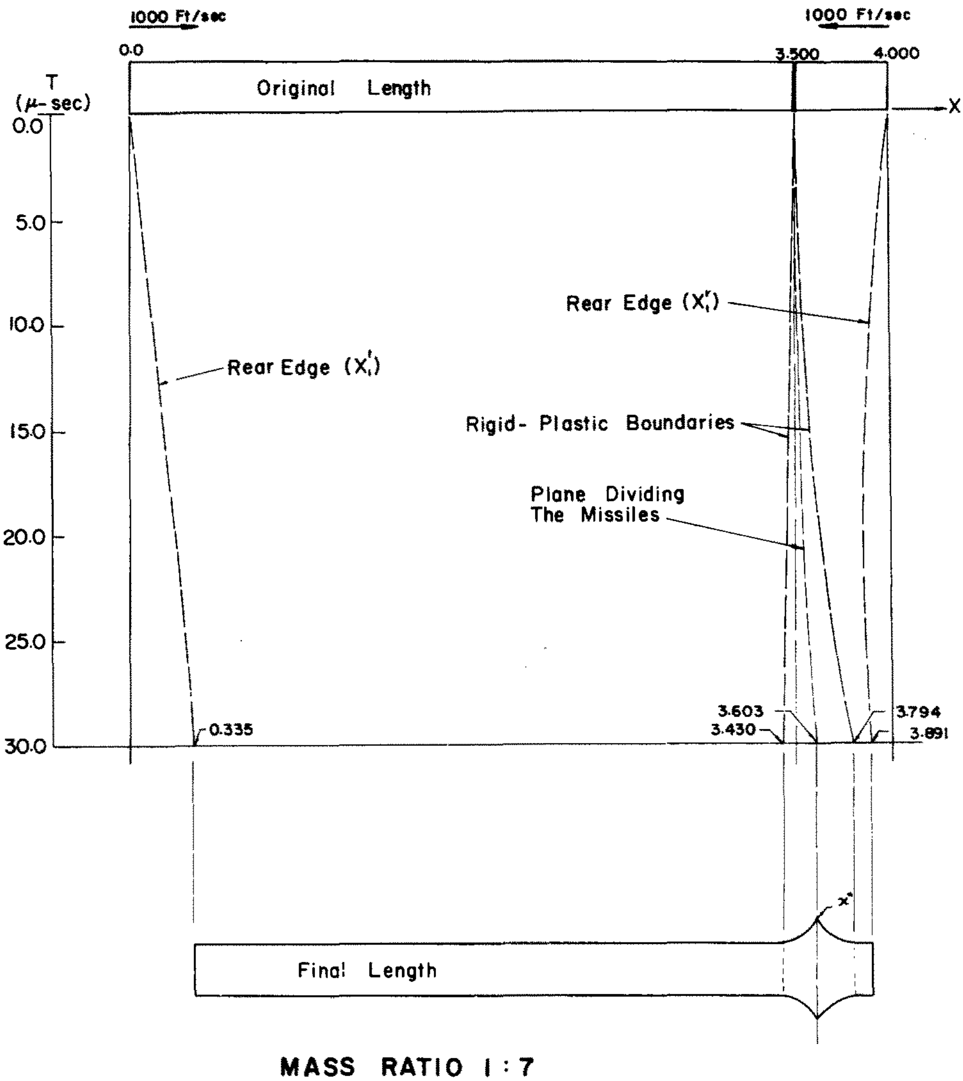

FIG. 9. X–T Diagram showing motion of (mass ratio 1 : 7): (a) The rigid–plastic boundaries; (b) The plane dividing the two missiles; (c) The rear ends of missiles.

Results of this analysis for unequal mass missiles were not compared since no experimental data has been found in literature. However, care was exercised to investigate that the total momentum of the system remained constant before, during, and after impact. In Fig. 7 it can be seen that the net momentum after impact was found to be the same as it was before impact. Also, it must be pointed out that the reaction force between the missiles was calculated, and was found to be equal and opposite during the process of impact. Thus, $F^l = F^r$.

In the case of Fig. 9 which shows total plastic deformation in a time interval of 30 μsec, justification for neglecting elastic effects might not be necessary. That is, if an elastic wave was assumed along with a plastic one a time $t = 2dX^1/c_e \simeq 35\,\mu$sec is needed for a single interaction of the elastic precursor in the longer missile. However, when shorter missiles are considered and an elasto–plastic analysis is contemplated an interaction of waves in the longer missile (see Fig. 8, $t = 2dX^1/c_e \simeq 10\,\mu$sec) would take place before impact is ended. Such an analysis is more difficult and needs different consideration than the one offered in this work. Therefore, it must be pointed out that in this paper by assuming a rigid, perfectly plastic model we have neglected the elastic precursors in the rigid parts of the missiles. In addition, the elastic wave which would be generated in the plastically-deformed part of missiles is also neglected. These assumptions were exercised and justified in Ref. [2] as well.

Further current work will be directed towards extension for missiles of different material, different cross-sectional area and different launch velocities. Generally, the analysis herein can be applied directly towards the study of penetration by realizing that if initially one of the projectiles has zero launch velocity, it can be considered to be the target of the other projectile.

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Абстракт—Настоящая работа развивает теорию большой пластической деформации металлических, цилиндрических, плоско законченых ракет, попадающих друг в друга с заданной скоростью удара. Результаты сравниваются с экспериментальными данными.

Группы сравнительных исследований, используемых для сравнения с настоящим методом следующие:

(а) Дж. И. Тейлор и А. Ц. Уиффин

(б) Э. Г. Ли, С. Т. Таппер, А. Ц. Уиффин и Г. Л. Д. Пюг

(в) Н. Дэвидс, Ц. Граберек, Д. Рафтопулос и А. Риччиацци.

На основе представленных сравнений результаты, полученные в этой работе, указывают, что теория описывает большие деформации вышеупомянутых ракет более точно для низких и средних скоростей.